1. $x + 2y = 9 \longrightarrow ①$

$xy + 18 = 0 \longrightarrow ②$

~~Equation~~ From ①, $x = 9 - 2y$

Replacing in ②.

$xy + 18 = 0$

$(9-2y)y + 18 = 0$

$9y - 2y^2 + 18 = 0$

$2y^2 - 9y - 18 = 0$ P: -36

$2y^2 - 12y + 3y + 18 = 0$ S: -9

$2y(y-6) + 3(y-6) = 0$ F: $-12, 3$

$(2y+3)(y-6) = 0$

$y = -\frac{3}{2}$ or $y = 6$

Replacing in $x = 9 - 2y$ $x = 9 - 2y$

$x = 9 - -3$ $x = 9 - 12$

$x = 12$ $x = -3$

coordinates of A and B = $\left(12, -\frac{3}{2}\right)$, $(-3, 6)$.

2. $2x^2 + 5x + 7$

$= 2\left[x^2 + \frac{5}{2}x + \frac{7}{2}\right]$

$= 2\left[\left(x + \frac{5}{4}\right)^2 - \frac{25}{16} + \frac{7}{2}\right]$

$= 2\left[\left(x + \frac{5}{4}\right)^2 + \frac{31}{16}\right]$

$= 2\left(x + \frac{5}{4}\right)^2 + \frac{31}{8}$ $\Rightarrow a = 2, b = \frac{5}{4}, c = \frac{31}{8}$

Minimum point = $\left(\frac{-5}{4}, \frac{31}{8}\right)$

3. $y = 2x + C \rightarrow$ 1

$y^2 = 4x \rightarrow$ 2

equating 1 and 2

$2x + C = \sqrt{4x}$

(squaring both sides)

$4x^2 + 4xC + C^2 = 4x$

$4x^2 + 4xC - 4x + C^2 = 0$

$4x^2 + (4C-4)x + C^2 = 0$

For tangent; $b^2 - 4ac = 0$

$(4C-4)^2 - 4(4)(C^2) = 0$

$\cancel{16C^2} - 32C + 16 - \cancel{16C^2} = 0$

$32C = 16$

$C = \frac{1}{2} \leftarrow$

4. $y = x^{\frac{1}{3}}$

$x^{\frac{2}{3}} - 5x^{\frac{1}{3}} + 6 = 0$

$y^2 - 5y + 6 = 0$ P: 6

$(y-2)(y-3) = 0$ S: −5

$y = 2$ or $y = 3$ F: −2, −3

$x^{\frac{1}{3}} = 2$ $x^{\frac{1}{3}} = 3$

$x = 8$ or $x = 27 \leftarrow$

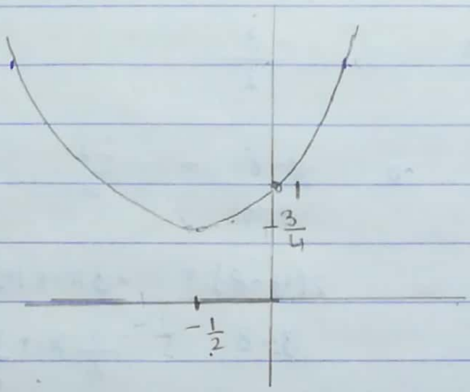
5. $f(x) = x^2 + x + 1$.

(i). $x^2 + x + 1 = 0$

$\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + 1 = 0$

$\left(x + \frac{1}{2}\right)^2 + \frac{3}{4} = 0$

min at $\left(-\frac{1}{2}, \frac{3}{4}\right)$.



→ f^{-1} does not exist since $f(x)$ is not one-one function .

(ii). when $x = -3$. when $x = 2$,

$f(x) = 7$ $f(x) = 7$

Answer: $\frac{3}{4} \leq f(x) \leq 7$.

(iii). Minimum value $= -\frac{1}{2}$

$x + \frac{1}{2} = 0$

$x = -\frac{1}{2}$ ←

6. (i) A (2,6), B (8,10) and C (6,0).

gradient of AB = $\frac{10-6}{8-2}$

$= \frac{4}{6}$

$= \frac{2}{3}$

Midpoint AB.

$= \left(\frac{2+8}{2} , \frac{6+10}{2} \right)$

$= (5, 8)$.

$\perp$ gradient $= \frac{-3}{2}$.

equation of $\perp$ bisector $\Rightarrow$ $\frac{y-8}{x-5} = \frac{-3}{2}$.

$2(y-8) = -3x + 15$.

$y-8 = \frac{-3}{2}x + \frac{15}{2}$

$y = -\frac{3}{2}x + \cancel{23}\ \frac{31}{2}$.

$2y = -3x + 31$

$3x + 2y = 31$

(i) gradient of BC = $\frac{10-0}{8-6}$

$= \frac{10}{2} = 5$.

Eq BC : $\frac{y-0}{x-6} = 5$

$y = 5x - 30 \longrightarrow BC \rightarrow 1$

$\perp$ bisector $\Rightarrow$ $y = -\frac{3}{2}x + \frac{\cancel{2}31}{2} \rightarrow 2$

$1 = 2$

$5x - 30 = -\frac{3}{2}x + \cancel{23}\ \frac{31}{2}$

(x2) $10x - 60 = -3x + \cancel{46}\ 31$

$13x = \cancel{106}\ 91$

$x = 7$

$y = 5x - 30$

$= 35 - 30$

$= 5$

D (7, 5) $\leftarrow$

7 (i) BD is perpendicular bisector of AC.

A $(-3,-4)$
C $(5,4)$.

midpoint of AC $= \left(\frac{-3+5}{2}, \frac{-4+4}{2}\right)$

$= (1, 0)$

gradient of AC $= \frac{4--4}{5--3}$

$= \frac{8}{8}$

$= 1$

$\perp$ gradient $= -1$.

Equation of BD $\Rightarrow$ $\frac{y-0}{x-1} = -1$

$y = -x+1$

$y = 1-x$. $\Leftarrow$

(ii). Equation of BC $\Rightarrow$ $\frac{y-4}{x-5} = 3$.

$y = 3x - 15 + 4$.

$y = 3x - 11$

Point where BD intersect BC = B.

BD = BC.

$1-x = 3x-11$

$3x+x = 1+11$

$4x = 12$

$x = 3$.

$y = 1-x$

$= 1-3$

$= -2$.

$\therefore B(3,-2) \leftarrow$

7.(ii) continued...

gradient of AD = gradient of BC

gradient of AD = 3..

Equation of AD = $\frac{y--4}{x--3} = 3$.

$y+4 = 3x+9$

$y = 3x+5$

Eqn of BD ⇒ $y = 1-x$.

AD = BD

$3x+5 = 1-x$.

$4x = -4$

$x = -1$

$y = 1-x$

$= 1--1$

$= 2$

D $(-1, 2)$ ←

(iii) A$(-3,-4)$

C$(5,4)$

$|AC| = \sqrt{(4--4)^2 + (5--3)^2}$

$= \sqrt{64+64}$

$= \sqrt{128}$

B$(3,-2)$

D$(-1,2)$.

$|BD| = \sqrt{(2--2)^2 + (-1-3)^2}$

$= \sqrt{16+16}$

$= \sqrt{32}$

$|AC| = 2 \times \sqrt{32}$

$= \sqrt{4} \times \sqrt{32}$

$= 2BD$ ←

(shown)

(iv). Area = $\frac{1}{2}\left| \begin{matrix} 5 & 3 & -3 & -1 & 5 \\ 4 & -2 & -4 & 2 & 4 \end{matrix} \right|$

$= \frac{1}{2}\left| (-10-12-6-4) - (12+6+4+10) \right|$

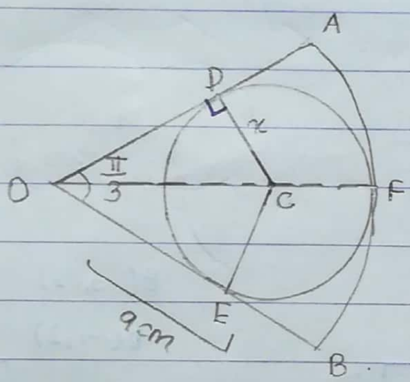
$= \frac{1}{2}\left| -32-32 \right|$

$= \frac{1}{2} \times 64$

$= 32$ square units.

↳ shown

8. (i)



$\sin C\hat{O}D = \frac{CD}{OC}$

$\sin\left(\frac{\pi}{6}\right) = \frac{x}{9-x}$

$\frac{1}{2} = \frac{x}{9-x}$

$2x = 9-x$

$3x = 9$

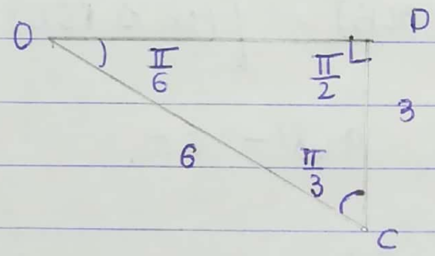
$x = 3$ cm ←

(ii). $OC = (9 - x)$

$= 9 - 3$

$= 6\text{cm}$

$DC = 3\text{cm}$.



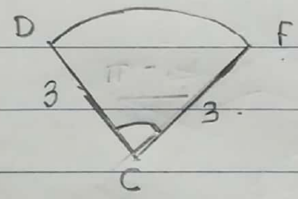
Area $= \frac{1}{2} \times a \times b \times \sin C$

$= \frac{1}{2} \times 6 \times 3 \times \sin\frac{\pi}{3}$

$= \frac{9}{2}\sqrt{3}\ \text{cm}^2 \leftarrow$

(shown).

(iii).



$D\hat{C}F = \dfrac{2\pi - \frac{2\pi}{3}}{2}$

$= \frac{2}{3}\pi$

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 9 \times \frac{2}{3}\pi$

$= 3\pi \leftarrow$

(shown)

(iv). Area of sector AOB = $\frac{1}{2} \times 9^2 \times \frac{\pi}{3}$

$= \frac{27}{2}\pi$.

Shaded Area = (Area of sector AOB) − [(2x △ COD) + (2 x sector DCF)]

$= \frac{27}{2}\pi - 9\sqrt{3} - 6\pi$

$= \frac{15}{2}\pi - 9\sqrt{3}$.

$= \frac{3}{2}(5\pi - 6\sqrt{3})$ cm3 ←

(shown)

9(i). $xy = 12$

$2x + y = k$

$k = 11$

$2x + y = 11$

$y = 11 - 2x$

$x(11 - 2x) = 12$.

$0 = 2x^2 - 11x + 12$.

$2x^2 - 11x + 12 = 0$

$2x^2 - 8x - 3x + 12 = 0$

$2x(x-4) - 3(x-4) = 0$

P: 24

S: −11

F: −8, −3

$x = \frac{3}{2}$, $x = 4$

$y = 11 - 2\left(\frac{3}{2}\right)$

$y = 11 - 3$

$y = 8$

$y = 11 - 2(4)$

$= 11 - 8$

$= 3$

$(4, 3)$, $\left(\frac{3}{2}, 8\right)$.

9(ii) $xy = 12$

$y = k - 2x$

$x(k-2x^2) = 12$

$kx - 2x^2 = 12.$

$2x^2 - kx + 12 = 0$

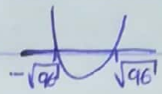
l does not intersect the curve

$b^2 - 4ac < 0$

$k^2 - 4(2)(12) < 0$

$k^2 - 96 < 0$

critical values = $\sqrt{96}$ and $-\sqrt{96}$



$-\sqrt{96} < k < \sqrt{96}$ ←

10.(i) $f(x) = 6(2x+3)^{-1}.$ $x \geqslant 0.$

~~f'(x)~~ $f'(x) = -6(2x+3)^{-2} \cdot 2$

$= \dfrac{-12}{(2x+3)^2}.$

since $(2x+3)^2 > 0$

$f'(x) < 0$

⇒ decreasing (shown).
function.

(ii). let $y = \dfrac{6}{2x+3}.$

$2xy + 3y = 6$

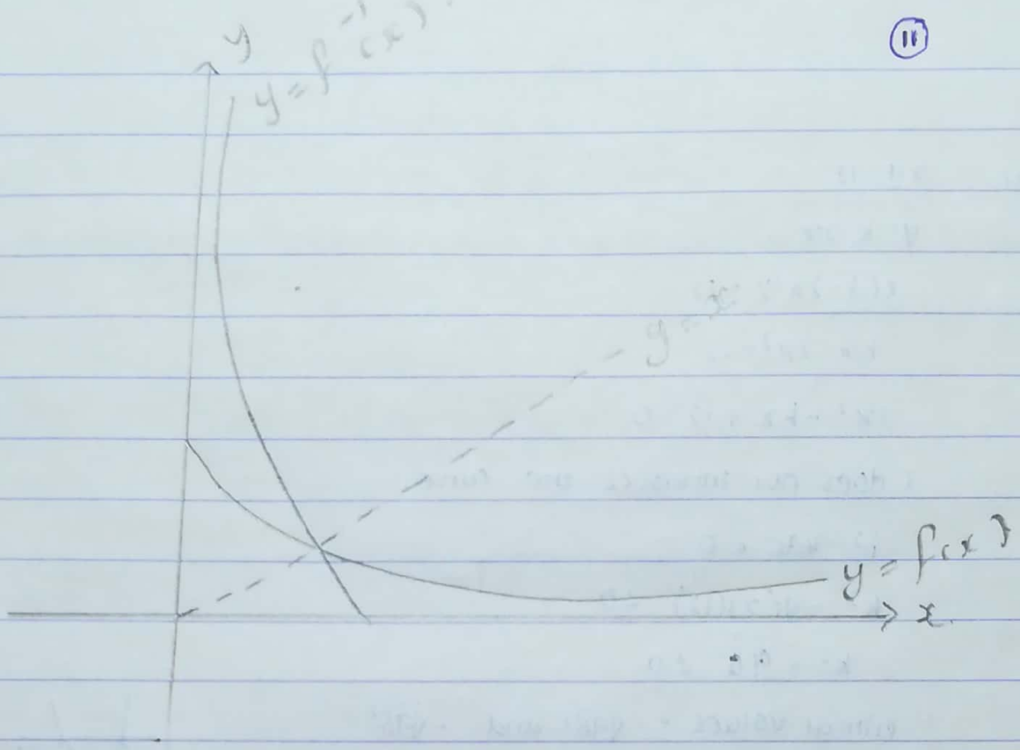
$2xy = 6 - 3y.$

$x = \dfrac{6-3y}{2y}.$

$f^{-1}(x) = \dfrac{6-3x}{2x}$

$x \geqslant 2.$

10 (iii).



(iv) $fg(x) = \frac{3}{2}$.

$$\frac{6}{2\left(\frac{1}{2}x\right)+3} = \frac{3}{2}$$

$$\frac{6}{x+3} = \frac{3}{2}.$$

$3x + 9 = 12$

$3x = 3$

$x = 1$ ←